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COMPUTATIONAL ASPECTS OF INCORPORATING AUXILIARY INFORMATION IN--ETC(U)

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Applied Research in Statistics - Mathematics - Operations Research

COMPUTATIONAL ASPECTS OF INCORPORATING AUXILIARY INFORMATION INTO AN IMPACT ACCELERATION INJURY PREDICTION MODEL.

by

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TECHNICAL REPORT ~~NO.~~ 112-7

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I. INTRODUCTION

A previous report [3] presented a general outline of procedures for simultaneously incorporating various sources of auxiliary information into an impact acceleration injury prediction model. This report, which serves as a companion volume, discusses computational aspects of the procedures. Specifically, the report presents computational procedures which allow the application of commonly used nonlinear estimation programs found in statistical packages such as BMDP [1] and SAS [2].

Throughout this report, reference will be made to equations in the previous report [3]. These equations, identified as (1) through (17) in that report, will be referenced herein by the same numbers as a matter of convenience. Equations in this report will therefore begin with reference number (18). In addition, the notation used in the previous report will be adopted without repeating its definition. Therefore, the reader may find it convenient (or perhaps necessary) to have that report readily available.

II. COMPUTATIONAL PROCEDURES

To estimate the model in (17) by using a statistical package such as SAS or BMDP, one should iteratively compute, for each observation:

(i) the model derivatives of (17) with respect to the parameters $\underline{\beta}_1$ and ρ

and (ii) the regression weights, $[p_i(1 - p_i)]^{-1}$.

The derivatives of (17) with respect to $\underline{\beta}_1$ and ρ are

$$\frac{\partial}{\partial \underline{\beta}_1} p_i = \phi(U_i(\underline{\beta}_1, \rho))(1 - \rho^2)^{-\frac{1}{2}} \underline{x}_i$$

$$\frac{\partial}{\partial \rho} p_i = \phi(U_i(\underline{\beta}_1, \rho)) \cdot \{-(1 - \rho^2)^{-\frac{1}{2}} s_i + [\rho/(1 - \rho^2)]U_i(\underline{\beta}_1, \rho)\}$$

where $\phi(U) = \frac{d}{dU} \Phi(U)$.

This paragraph concludes the discussion of the problem without regard to prior information.

A. PRIOR INFORMATION

The model parameters in (6) or (7), as the case may be, can be estimated conveniently and efficiently by using the model in (17) to incorporate the prior information into the parameter estimates. Here again the intuition behind this approach comes from the likelihood of the observations. Consider the likelihood in (12). The form of this likelihood suggests the models:

$$y_{1i} = \Phi[(1 - \rho^2)^{-\frac{1}{2}}(\underline{x}_i'\underline{\beta}_1 - \rho s_i)] + \xi_i \tag{18a}$$

$$y_{2i} = \underline{x}_i'\underline{\beta}_2 + \varepsilon_{2i} \tag{18b}$$

where $E(\xi_i) = 0$, $\mathrm{Var}\ (\xi_i) = p_i(1 - p_i)$, $p_i = \Phi[U_i(\underline{\beta}_1, \rho)]$

$E(\varepsilon_{2i}) = 0$, $\mathrm{Var}\ (\varepsilon_{2i}) = \sigma_2^2$.

Note that the ξ_i and ε_{2i} error terms are stochastically independent, since y_{1i}/s_i and y_{2i} are independent. This fact makes the analysis by statistical packages convenient since their nonlinear regression algorithms cannot analyze correlated observations directly.

As stated in the previous report [3], it is assumed that the prior information can be in the form of

(i) a priori estimates of *some function of* the model parameters

and/or (ii) a priori knowledge in the form of model parameter equality constraints.

The technique used to model the prior information discussed in the previous report can be applied to the models in (18a) and (18b) to form an overall model of direct and auxiliary information.

Let the prior estimates of some function of the model parameters be expressed by (6), where

$$\underline{\alpha} = \begin{pmatrix} \beta \\ \rho \end{pmatrix} .$$

Recall that it is assumed that the prior information embodied in $\underline{r}$ is stochastically independent of

$$\begin{pmatrix} y_{11} \\ y_{21} \end{pmatrix} , \ldots\ldots , \begin{pmatrix} y_{1n} \\ y_{2n} \end{pmatrix} .$$

$$\text{Let } \underline{f}_i(\underline{\beta}, \rho) = \begin{bmatrix} f_{1i}(\underline{\beta}, \rho) \\ \\ f_{2i}(\underline{\beta}, \rho) \end{bmatrix} = \begin{bmatrix} \Phi(U_i(\underline{\beta}_1, \rho)) \\ \\ \underline{x}_i'\underline{\beta}_2 \end{bmatrix}, \underline{f}(\underline{\beta}, \rho) = \begin{bmatrix} \underline{f}_1(\underline{\beta}, \rho) \\ \vdots \\ \underline{f}_n(\underline{\beta}, \rho) \end{bmatrix}$$

$$\underline{\eta}_i = \begin{bmatrix} \xi_i \\ \\ \varepsilon_{2i} \end{bmatrix}, \quad \underline{\eta} = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_n \end{bmatrix} .$$

Then all the information, barring parameter constraints, can be modeled as

$$\underline{r} = \underline{g}(\underline{\beta}, \rho) + \underline{\nu} \tag{19a}$$

$$\underline{y} = \underline{f}(\underline{\beta}, \rho) + \underline{\eta} \tag{19b}$$

where $E(\underline{\nu}) = 0$, $Var(\underline{\nu}) = \underline{\Psi}$

and $E(\underline{\eta}) = 0$, $Var(\underline{\eta}) = \underline{V}^* = \begin{bmatrix} \underline{V}_1^* & & 0 \\ & \ddots & \\ 0 & & \underline{V}_n^* \end{bmatrix}$, $\underline{V}_i^* = \begin{bmatrix} p_i(1 - p_i) & 0 \\ \\ 0 & \sigma_2^2 \end{bmatrix}$,

$$p_i = \Phi(U_i(\underline{\beta}_1, \rho)) .$$

The vector $\underline{r}$ is conceptually and computationally considered as an observation. Due to the structure of most statistical packages, this concept can be computationally exploited only if $\underline{\Psi}$ is a diagonal matrix, where the diagonal elements of $\underline{\Psi}$ are considered as weights for the observations in $\underline{r}$. Even if $\underline{\Psi}$ is a diagonal matrix, it is probably the easiest for the user if he or she transforms the elements of $\underline{r}$ so that the transformed elements have the identity matrix as covariance matrix. This is done in this report.

The elements of $\underline{r}$ can be transformed by premultiplying $\underline{r}$ by a matrix $\underline{\Gamma}$ such that

$$\underline{\Gamma}'\underline{\Gamma} = \underline{\Psi}^{-1} .$$

If $\underline{r}^*$ denotes the transformed elements of $\underline{r}$, then (19a) can be reexpressed as

$$\underline{r}^* = \underline{\Gamma r} = \underline{g}^*(\underline{\beta}, \rho) + \underline{\Gamma \nu}$$

where $$\underline{g}^*(\underline{\beta}, \rho) = \underline{\Gamma g}(\underline{\beta}, \rho)$$

and $$E(\underline{\Gamma \nu}) = 0, \ \mathrm{Var}(\underline{\Gamma \nu}) = \underline{\Gamma}'(\underline{\Gamma}'\underline{\Gamma})^{-1}\underline{\Gamma}' = \underline{I}.$$

B. PARAMETER CONSTRAINTS

To include constraints on the parameters, let

$$(\underline{\beta}, \rho) = \underline{h}(\underline{\theta}) \qquad (20)$$

where $$\dim \underline{\theta} \ \underline{<} \ \dim(\underline{\beta}, \rho).$$

Then, it follows that, defining $\underline{\nu}^* = \underline{\Gamma\nu}$,

$$\underline{r}^* = \underline{g}^*(\underline{h}(\underline{\theta})) + \underline{\nu}^* \tag{21a}$$

$$\underline{y} = \underline{f}(h(\underline{\theta})) + \underline{\eta} \quad . \tag{21b}$$

Note that indicator variables can be used to facilitate the computation of the estimates of $(\underline{\beta}, \rho)$ with a statistical package.

For example, consider the case without constraints and let

$$z_i = \delta_{1i}y_{1i} + \delta_{2i}y_{2i} + (1 - \delta_{1i})(1 - \delta_{2i})r_j^* \tag{22}$$

$$k_i(\underline{\beta}, \rho) = \delta_{1i}f_{1i}(\underline{\beta}, \rho) + \delta_{2i}f_{2i}(\underline{\beta}, \rho) + (1 - \delta_{1i})(1 - \delta_{2i})g_j^*(\underline{\beta}, \rho)$$

$$e_i = \delta_{1i}\xi_i + \delta_{2i}\varepsilon_{1i} + (1 - \delta_{1i})(1 - \delta_{2i})\nu_j^*$$

where $$\delta_{1i} = \begin{cases} 1 \text{ if } z_i = y_{1i},\ k_i(\underline{\beta}, \rho) = f_{1i}(\underline{\beta}, \rho), \text{ and } e_i = \xi_i \\ 0 \text{ otherwise} \end{cases}$$

$$\delta_{2i} = \begin{cases} 1 \text{ if } z_i = y_{2i},\ k_i(\underline{\beta}, \rho) = f_{2i}(\underline{\beta}, \rho), \text{ and } e_i = \varepsilon_{2i} \\ 0 \text{ otherwise} \end{cases}$$

and $i = 1,\ldots,n$, $j = 1,\ldots,m$, $m = \dim \underline{r}^*$.

Here r_j, g_j^*, and ν_j are the j^{th} elements of $\underline{r}$, $\underline{g}^*$, and $\underline{\nu}$ respectively.

Now rewrite (22) as

$$\underline{z} = \underline{k}(\underline{\beta}, \rho) + \underline{e} \tag{23}$$

where $\underline{z} = \begin{bmatrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_n \end{bmatrix}$, $\underline{k}(\underline{\beta}, \rho) = \begin{bmatrix} k_1(\underline{\beta}, \rho) \\ \cdot \\ \cdot \\ \cdot \\ k_n(\underline{\beta}, \rho) \end{bmatrix}$, $\underline{e} = \begin{bmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{bmatrix}$

and $E(\underline{e}) = 0$, $\mathrm{Var}(e_i) = \delta_{1i}p_i(1 - p_i) + \delta_{2i}\sigma_2^2 + (1 - \delta_{1i})(1 - \delta_{2i})$, with $e_1,\ldots,e_n$ independent.

Next, if parameter constraints exist as in (20), just substitute $\underline{h}(\underline{\theta})$ for $(\underline{\beta}, \rho)$ in the model in (23). The above indicator variables will enable the statistical package to directly analyze all the information modeled in (23) via standard nonlinear, least squares algorithms. One merely needs to iteratively compute:

(i) model derivatives of (23) with respect to $(\underline{\beta}, \rho)$ or $\underline{\theta}$ if constraints are to be used,

and (ii) the regression weights $[\mathrm{Var}(e_i)]^{-1}$.

III. A POSSIBLE EXPERIMENT

To elucidate the application of estimating the model in (23), an example of a possible impact acceleration injury experiment with n observations may be considered. Suppose that for $i = 1,\ldots,n$:

(i) $\underline{x}_i = (1, x_{1i})'$ where x_{1i} denotes peak sled acceleration for the i^{th} subject,

(ii) y_{1i} is a dichotomous injury observation for i^{th} subject, with value of 1 for injury, 0 for no injury,

and (iii) y_{2i} is a continuous preinjury measurement for the i^{th} subject. Then the model of the empirical data as given in (19b) has

$$\underline{\beta} = \begin{bmatrix} \underline{\beta}_1 \\ \underline{\beta}_2 \end{bmatrix}, \quad \underline{\beta}_1 = \begin{bmatrix} \beta_{01} \\ \beta_{11} \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} \beta_{02} \\ \beta_{12} \end{bmatrix}$$

A. SOME ASSUMPTIONS

Suppose further, that for a particular level of peak sled acceleration, x_0 say, a good prior unbiased estimate of injury probability, p_0, is available. Let this prior injury probability and peak sled acceleration level be related by

$$p_0 = \Phi(\beta_{01} + \beta_{11}x_0).$$

Assume that the variance of p_0 is known or estimated to be ψ .

Thus, the prior information as modeled in (19a) can be written as

$$\underline{r}^* = \underline{g}^*(\underline{\beta}, \rho) + \underline{\Gamma\nu}$$

where $\underline{\Gamma}$ is a 1x1 matrix estimated by ψ^{-1}, $\underline{r}^*$ is a 1x1 vector equal to $\psi^{-1}p_0$, and $\underline{g}^*(\underline{\beta}, \rho) = \psi^{-1}[\Phi([1, x_0, 0, 0]\underline{\beta})]$.

Finally, assume that the elements of $\underline{\beta}$ are constrained by the equation $\beta_{12} = \sigma_2\beta_{11}$. This constraint can arise quite naturally from the following situation. Suppose that the event of a preinjury measurement, (e.g., change in evoked potential response) exceeding some critical, prespecified value, y_0, is always at least as likely as the occurrence of an injury, for all levels of peak sled acceleration. This can be mathematically represented by

$$\Pr(y_{2i} > y_0|\underline{x}) \geq \Pr(y_{1i} = 1|\underline{x}) \text{ for all } \underline{x}.$$

Now,

$$\Pr(y_{2i} > y_0|\underline{x}) = \Phi[-(y_0 - \underline{x}'\underline{\beta}_2)/\sigma_2]$$

and $\Pr(y_{1i} = 1|\underline{x}) = \Phi(\underline{x}'\underline{\beta}_1)$.

Therefore, for all $\underline{x}$,

$$\Phi[-(y_0 - \underline{x}'\underline{\beta}_2)/\sigma_2] \geq \Phi(\underline{x}'\underline{\beta}_1).$$

However, this can only be true if

$$\beta_{12}/\sigma_2 = \beta_{11} .$$

The constraint imposed by the vector valued function $\underline{h}$ can be described by

$$\underline{\beta} = \begin{bmatrix} \beta_{01} \\ \beta_{11} \\ \hline \beta_{02} \\ \beta_{12} \\ \rho \end{bmatrix} = \underline{h}(\underline{\theta}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underline{\theta} ,$$

where

$$\underline{\theta} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} \beta_{01} \\ \beta_{11} \\ \beta_{02} \\ \rho \end{bmatrix} .$$

However, $\underline{h}$ depends upon σ_2, which is unknown. In other words, the prior information, in terms of the parameter constraints, is not complete. In this particular case, the best that can be done is to estimate $\underline{h}$ by $\hat{\underline{h}}$, where

$$\hat{\underline{h}}(\underline{\theta}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \tilde{\sigma}_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underline{\theta}$$

and $\tilde{\sigma}_2$ is the minimum variance, unbiased estimator of σ_2 based on the preinjury data, i.e. $\tilde{\sigma}_2 = \frac{n}{n-2}\hat{\sigma}_2$, $\hat{\sigma}_2$ being the maximum likelihood estimate of σ_2.

B. MODEL REPRESENTATION

The model representation for computer analysis is then given (for $i = 1,\ldots,n$ and $j = 1$) by:

$$z_i = \delta_{1i}y_{1i} + \delta_{2i}y_{2i} + (1 - \delta_{1i})(1 - \delta_{2i})r_j^*$$

$$k_i(\underline{\beta}, \rho) = \delta_{1i}f_{1i}(\hat{\underline{h}}(\underline{\theta}) + \delta_{2i}f_{2i}(\hat{\underline{h}}(\underline{\theta})) + (1 - \delta_{1i})(1 - \delta_{2i})g_j^*(\hat{\underline{h}}(\underline{\theta}))$$

$$e_i = \delta_{1i}\xi_i + \delta_{2i}\varepsilon_{2i} + (1 - \delta_{1i})(1 - \delta_{2i})\nu_j^*$$

where δ_{1i} and δ_{2i} were defined previously, and

$$\begin{bmatrix} f_{1i}(\hat{\underline{h}}(\underline{\theta})) \\ \\ f_{2i}(\hat{\underline{h}}(\underline{\theta})) \end{bmatrix} = \begin{bmatrix} \Phi(U_i(\underline{\beta}_1, \rho)) \\ \\ \beta_{02} + x_{1i}\tilde{\sigma}_2\beta_{11} \end{bmatrix} ,$$

$$r_j^* = \psi^{-1}p_0 \quad ,$$

$$g_j^*(\hat{\underline{h}}(\underline{\theta})) = \psi^{-1}\Phi(\beta_{01} + x_0\beta_{11}).$$

If a separate estimate of $\underline{\beta}_1$ based on the dichotomous injury data and separate estimates of $\underline{\beta}_2$ and σ_2 based on the continuous preinjury data are available, then a "quick-and-dirty" estimate of ρ can be computed. Note that

$$E(\varepsilon_{2i}y_{1i}) = -\rho\sigma_2\phi(\underline{x}_i'\underline{\beta}_1) \tag{24}$$

$$Var(\varepsilon_{2i}y_{1i}) = \sigma_2^2\{\Phi(\underline{x}_i'\underline{\beta}_1) - \rho^2[(\underline{x}_i'\underline{\beta}_1)\phi(\underline{x}_i'\underline{\beta}) + \phi^2(\underline{x}_i'\underline{\beta}_1)]\}$$

The results in (24) are derived in the appendix of this and the previous technical report [3]. From (24) it is easy to see that

$$E\{-\varepsilon_{2i}y_{1i}(\sigma_2\phi(\underline{x}_i'\underline{\beta}_1))^{-1}\} = \rho \ .$$

Thus,

$$-\sum_{i=1}^{n} \varepsilon_{2i}y_{1i}/[\phi(\underline{x}_i'\tilde{\underline{\beta}}_1)n\tilde{\sigma}_2] \tag{25}$$

is an approximately unbiased estimate of ρ. Here $\tilde{\underline{\beta}}_1$ is the probit estimate of $\underline{\beta}_1$ based on the dichotomous injury data only and $\tilde{\sigma}_2^2$ is the unbiased estimate of σ_2^2,

$$\frac{1}{(n-2)}\sum_{i=1}^{n} \varepsilon_{2i}^2 \ .$$

Unfortunately, the estimate in (25) can sometimes have absolute values exceeding one. A more refined (approximately unbiased) estimate of ρ can be easily obtained by using an iterative least squares statistical algorithm. To compute an iteratively reweighted, least squares estimate of ρ, simply let

$$\begin{bmatrix} \varepsilon_{21}y_{11} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{2n}y_{1n} \end{bmatrix} \tag{26}$$

be the dependent observations and let

$$\begin{bmatrix} -\tilde{\sigma}_2\phi(\underline{x}_1'\tilde{\underline{\beta}}_1) \\ \cdot \\ \cdot \\ \cdot \\ -\tilde{\sigma}_2\phi(\underline{x}_n'\tilde{\underline{\beta}}_1) \end{bmatrix} \tag{27}$$

be the independent observations. Compute the initial weights from the reciprocals of the variances of $\varepsilon_{2i}y_{1i}$ by picking an initial value for ρ. The model derivatives are estimated by (27).

C. CORRELATION

Thus far, all models have assumed that ρ is constant for all $\underline{x}$. An approximate chi-square test can be conducted to test the null hypothesis that ρ does not depend upon $\underline{x}$. From (24) the means and variances of $\varepsilon_{2i}y_{1i}$ can be computed. Let

$$\mu_i = E(\varepsilon_{2i} y_{1i}) ,$$

$$\sigma_i = \mathrm{Var}(\varepsilon_{2i} y_{1i}) ,$$

$$w_i = \frac{\varepsilon_{2i} y_{2i} - \mu_i}{\sigma_i} ,$$

$$\text{and } \bar{w} = \frac{1}{n} \sum_{i=1}^{n} w_i .$$

Then $w_1, \ldots, w_n$ are stochastically independent, each with mean zero and variance equal to 1. It follows that

$$\sum_{i=1}^{n} (w_i - \bar{w})^2$$

has an approximate chi-square distribution with (n - 1) degrees of freedom. Of course all the μ_i's and σ_i's are not known, but they can be estimated by substituting the estimates of the model parameters via the equations in (24). Replacing the μ_i's and σ_i's by their estimates should still yield a reasonably powerful test against significant departures from the null hypothesis for moderate to large samples sizes.

If the chi-square test rejects the null hypothesis, then it is still possible to estimate a probit model where ρ depends upon $\underline{x}$. However in this case, it would be desirable to increase the sample size. This is due to the addition of the extra parameters used to model ρ . The computation of the model parameter estimates would also be considerably more difficult to obtain.

IV. DISCUSSION

The purpose of using auxiliary information is to improve the estimate of $\underline{\beta}_1$. In some estimation situations, however, the use of auxiliary information may result in estimates of $\underline{\beta}_1$ that are slightly less accurate than if the auxiliary information were not used at all. If the constraint information is correct and the prior estimates of $\underline{\beta}_1$ are close to the true value of $\underline{\beta}_1$, then the a priori auxiliary information should always contribute to reducing the mean square error (MSE) of $\underline{\beta}_1$.

If, however, the sample size is small, there is no apriori information linking the parameters of $\underline{\beta}_1$ with $\underline{\beta}_2$, and in addition the correlation between the injury tolerance and the side effect (i.e., preinjury) is low, then the incorporation of preinjury data could possibly contribute to a slight increase in the MSE of the estimate of $\underline{\beta}_1$. In order to assess the benefits of the inclusion of preinjury data for small sample sizes, a Monte Carlo study is being conducted. This study will be discussed in a future report.

V. REFERENCES

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[2] Helwig, J. and Council, K., eds., SAS User's Guide, SAS Institute, Inc., Raleigh, NC, 1979.

[3] Peterson, J. J. and Smith, D. E., "A General Statistical Approach for Using Auxiliary Information in the Development of an Impact Acceleration Injury Prediction Model," Technical Report No. 112-6, Desmatics, Inc., August 1980.

APPENDIX

Since

$$\mathrm{Var}(\varepsilon_{2i}y_{1i}) = E\{(\varepsilon_{2i}y_{1i})^2\} - \{E(\varepsilon_{2i}y_{1i})\}^2,$$

it is sufficient, in view of the appendix in [3], to derive an expression for $E\{(\varepsilon_{2i}y_{1i})^2\}$.

Now,

$$E\{(\varepsilon_{2i}y_{1i})^2\} = E\{\varepsilon_{2i}^2 y_{1i}\}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varepsilon_{2i}^2 I_i(t_i) f_{\varepsilon,T}(\varepsilon_{2i}, t_i)\, d\varepsilon_{2i}\, dt_i$$

$$= \int_{-\infty}^{0}\int_{-\infty}^{\infty} \varepsilon_{2i}^2 f_\varepsilon(\varepsilon_{2i}|t_i) f_T(t_i)\, d\varepsilon_{2i}\, dt_i$$

$$= \int_{-\infty}^{0} E\{\varepsilon_{2i}^2|t_i\} f_T(t_i)\, dt_i$$

However,

$$E\{\varepsilon_{2i}^2|t_i\} = \mathrm{Var}(\varepsilon_{2i}|t_i) + \{E(\varepsilon_{2i}|t_i)\}^2$$

$$= (1 - \rho^2)\sigma_2^2 + \rho^2\sigma_2^2(t_i - \mu_{T_i})^2/\sigma_T^2$$

since $\varepsilon_{2i}|t_i$ has a normal distribution with mean

$$\rho\sigma_2(t_i - \mu_{T_i})/\sigma_T$$

and variance

$$(1 - \rho^2)\sigma_2^2 .$$

Thus,

$$E\{(\varepsilon_{2i}y_{1i})^2\} = (1 - \rho^2)\sigma_2^2\int_{-\infty}^{0} f_T(t_i)dt_i + (\rho^2\sigma_2^2/\sigma_\tau^2)\int_{-\infty}^{0}(t_i - \mu_{T_i})^2 f_T(t_i)dt_i \ .$$

Now, from (3) and (4) in the appendix of [3],

$$\int_{-\infty}^{0} f_T(t_i)dt_i = \int_{-\infty}^{-\mu_{T_i}/\sigma_\tau} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$$

$$= \Phi(-\mu_{T_i}/\sigma_\tau)$$

$$= \Phi(\underline{x}_i'\underline{\beta}_1) \ .$$

Also,

$$\int_{-\infty}^{0} [(t_i - \mu_{T_i})^2/\sigma_\tau^2] f_{T_i}(t_i)dt_i = \int_{-\infty}^{-\mu_{T_i}/\sigma_\tau} u^2 \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du \ .$$

By performing integration by parts, the integral on the right hand side becomes

$$-u(2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}u^2} \Big|_{-\infty}^{-\mu_{T_i}/\sigma_\tau} + \int_{-\infty}^{-\mu_{T_i}/\sigma_\tau} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

This expression is equal to

$$-(\mu_{T_i}/\sigma_\tau)\phi(-\mu_{T_i}/\sigma_\tau) + \Phi(\mu_{T_i}/\sigma_\tau) = -(\underline{x}_i'\underline{\beta}_1)\phi(\underline{x}_i'\underline{\beta}_1) + \Phi(\underline{x}_i'\underline{\beta}_1)$$

Therefore,

$$E\{(\varepsilon_{2i}y_{1i})^2\} = (1 - \rho^2)\sigma_2^2\Phi(\underline{x}_i'\underline{\beta}_1) + \rho^2\sigma_2^2[\Phi(\underline{x}_i'\underline{\beta}_1) - (\underline{x}_i'\underline{\beta}_1)\phi(\underline{x}_i'\underline{\beta}_1)] \ ,$$

and hence,

$$\mathrm{Var}(\varepsilon_{2i}y_{1i}) = \sigma_2^2[\Phi(\underline{x}_i'\underline{\beta}_1) - \rho^2((\underline{x}_i'\underline{\beta}_1)\phi(\underline{x}_i'\underline{\beta}_1) + \phi^2(\underline{x}_i'\underline{\beta}_1))] \ .$$

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Auxiliary information may sometimes be used in development of a mathematical model in order to improve the estimated values of unknown parameters. This report discusses computational procedures which allow the application of commonly used nonlinear estimation programs to incorporate various sources of auxiliary information into an impact acceleration injury prediction model.

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